Classification of the Equilibrium Point (0, 0) of the Linear Planar System $X' = A\,X$ with $\det A \neq 0$

Let λ_1 and λ_2 be the eignevalues of A.

1. λ_1 and λ_2 are real-valued and $\lambda_1 \neq \lambda_2$.
   - (i) $\lambda_1 < \lambda_2 < 0$: (0, 0) is a sink (stable node).
   - (ii) $0 < \lambda_1 < \lambda_2$: (0, 0) is a source (unstable node).
   - (iii) $\lambda_1 < 0 < \lambda_2$: (0, 0) is a saddle point (col).
2. $\lambda_1 = \lambda_2$.
   - (a) A has two linearly independent eigenvectors.
     - (i) $\lambda_1 \leq \lambda_2 < 0$: (0, 0) is a sink (stable singular node).
     - (ii) $0 < \lambda_1 \leq \lambda_2$: (0, 0) is a source (unstable singular node).
   - (b) A has only one linearly independent eigenvector.
     - (i) $\lambda_1 \leq \lambda_2 < 0$: (0, 0) is a sink (stable degenerate node).
     - (ii) $0 < \lambda_1 \leq \lambda_2$: (0, 0) is a source (unstable degenerate node).
3. λ_1 and λ_2 are complex-valued: $\lambda_1, \lambda_2 = \alpha \pm i\beta$ with $\beta \neq 0$.
   - (i) $\mathrm{Re}\,\lambda_1 = \mathrm{Re}\,\lambda_2 = \alpha < 0$: (0, 0) is a spiral sink (stable focus).
   - (ii) $\mathrm{Re}\,\lambda_1 = \mathrm{Re}\,\lambda_2 = \alpha > 0$: (0, 0) is a spiral source (unstable focus).
   - (iii) $\mathrm{Re}\,\lambda_1 = \mathrm{Re}\,\lambda_2 = \alpha = 0$: (0, 0) is a center.

In short, for all cases,

- (i) If both $\mathrm{Re}\,\lambda_1$ and $\mathrm{Re}\,\lambda_2$ are less than zero, then (0, 0) is a sink (stable critical point).
- (ii) If both $\mathrm{Re}\,\lambda_1$ and $\mathrm{Re}\,\lambda_2$ are greater than zero, then (0, 0) is a source (unstable critical point).
- (iii) If $\mathrm{Re}\,\lambda_1$ and $\mathrm{Re}\,\lambda_2$ have opposite signs, then (0, 0) is a saddle point.
  If $\mathrm{Re}\,\lambda_1$ is zero but $\mathrm{Im}\,\lambda_1$ is not zero, then (0, 0) is a center.